

The study of leading twist light cone wave functions of J/Ψ meson.

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This paper is devoted to the study of leading twist light cone wave functions of J/Ψ meson. The moments of these wave functions have been calculated within three approaches: potential models, nonrelativistic QCD and QCD sum rules. Using the results obtained within these approaches the models for the light cone wave functions of leading twist have been proposed. Similarly to the wave function of η_c meson the leading twist light cone wave functions of J/Ψ meson have very interesting properties at scales $\mu > m_c$: improvement of the accuracy of the model, appearance of relativistic tail and violation of nonrelativistic QCD velocity scaling rules. The last two properties are the properties of true leading twist light cone wave functions of J/Ψ meson.

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I. INTRODUCTION

Commonly exclusive charmonium production at high energies is studied within nonrelativistic QCD (NRQCD) [1]. In the framework of this approach charmonium is considered as a bound state of quark-antiquark pair moving with small relative velocity $v \ll 1$. Due to the presence of small parameter v the amplitude of charmonium production can be built as an expansion in relative velocity v .

Thus in the framework NRQCD the amplitude of any process is a series in relative velocity v . Usually, in the most of applications of NRQCD, the consideration is restricted by the leading order approximation in relative velocity. However, this approximation has two problems which make it unreliable. The first problem is connected with rather large value of relative velocity for charmonium: $v^2 \sim 0.3$, $v \sim 0.5$. For this value of v^2 one can expect large contribution from relativistic corrections in any process. So in any process resummation of relativistic corrections should be done or one should prove that resummation of all terms is not crucial. The second problem is connected with QCD radiative corrections. The point is that due to the presence of large energy scale Q there appears large logarithmic terms $(\alpha_s \log Q/m_c)^n$, $Q \gg m_c$ which can be even more important than relativistic corrections at sufficiently large energy ($Q \sim 10$ GeV). So these terms should also be resummed. In principle, it is possible to resum large logarithms in the NRQCD factorization framework [2, 3], however such resummation is done rarely.

The illustration of all mentioned facts is the process of double charmonium production in e^+e^- annihilation at B-factories, where leading order NRQCD predictions [4, 5, 6] are approximately by an order of magnitude less than experimental results [7, 8]. The calculation of QCD radiative corrections [9] diminished this disagreement but did not remove it. Probably the agreement with the experiments can be achieved if, in addition to QCD radiative corrections, relativistic corrections will be resummed [10].

In addition to NRQCD, hard exclusive processes can be studied in the framework of light cone expansion formalism [11, 12] where both problems mentioned above can be solved. Within light cone expansion formalism the amplitude is built as an expansion over inverse powers of characteristic energy of the process. Usually this approach is successfully applied to exclusive production of light mesons [12]. However recently the application of light cone expansion formalism to double charmonium production [13, 14, 15, 16] allowed one to achieve good agreement with the experiments.

In the framework of light cone formalism the amplitude of some meson production in any hard process can be written as a convolution of the hard part of the process, which can be calculated using perturbative QCD, and process independent light cone wave function (LCWF) of this meson that parameterizes nonperturbative effects. From this one can conclude that charmonium LCWFs are key ingredients of any hard exclusive process with charmonium production. In paper [17] the leading twist light cone wave function of η_c meson was studied. This paper is devoted to the study of leading twist LCWFs of J/Ψ meson.

The paper is organized as follows. In the next section all definitions needed in our calculation will be given. In Section III the moments of LCWFs will be calculated in the framework of Buchmuller-Tye and Cornell potential models. Section IV is devoted to the calculation of the moments within NRQCD. QCD sum rules will be applied to the calculation of the moments in Section V. Using the results obtained in Sections III-V the models of LCWFs will be built in Section VI. In the last section the results of this paper will be summarized.

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II. DEFINITIONS.

There are two leading twist light cone wave functions (LCWF) of J/Ψ meson. The first one is twist 2 LCWF of longitudinally polarized J/Ψ meson $\phi_L(\xi, \mu)$. The second one is twist 2 LCWF of transversely polarized J/Ψ meson $\phi_T(\xi, \mu)$. These LCWFs can be defined as follows [12]

$$\begin{aligned} \langle 0 | \bar{Q}(z) \gamma_\alpha [z, -z] Q(-z) | J/\Psi(\epsilon_{\lambda=0}, p) \rangle_\mu &= f_L p_\alpha \int_{-1}^1 d\xi e^{i(pz)\xi} \phi_L(\xi, \mu), \\ \langle 0 | \bar{Q}(z) \sigma_{\alpha\beta} [z, -z] Q(-z) | J/\Psi(\epsilon_{\lambda=\pm 1}, p) \rangle_\mu &= f_T(\mu) (\epsilon_\alpha p_\beta - \epsilon_\beta p_\alpha) \int_{-1}^1 d\xi e^{i(pz)\xi} \phi_T(\xi, \mu), \end{aligned} \quad (1)$$

where the following designations are used: x_1, x_2 are the parts of momentum of the whole meson carried by quark and antiquark correspondingly, $\xi = x_1 - x_2$, p is a momentum of J/Ψ meson, μ is an energy scale. The factor $[z, -z]$, that makes matrix element (1) gauge invariant, is defined as

$$[z, -z] = P \exp \left[ig \int_{-z}^z dx^\mu A_\mu(x) \right]. \quad (2)$$

The LCWFs $\phi_{L,T}(\xi, \mu)$ are normalized as

$$\int_{-1}^1 d\xi \phi_{L,T}(\xi, \mu) = 1. \quad (3)$$

With this normalization condition the constants $f_{T,L}$ are defined as

$$\begin{aligned} \langle 0 | \bar{Q}(0) \gamma_\alpha Q(0) | J/\Psi(\epsilon_{\lambda=0}, p) \rangle &= f_L \epsilon_{\lambda=0}, \\ \langle 0 | \bar{Q}(0) \sigma_{\alpha\beta} Q(0) | J/\Psi(\epsilon_{\lambda=\pm 1}, p) \rangle_\mu &= f_T(\mu) (\epsilon_\alpha p_\beta - \epsilon_\beta p_\alpha). \end{aligned} \quad (4)$$

It should be noted here that the local current $\bar{Q}(0) \gamma_\alpha Q(0)$ is renormalization group invariant. The local current $\bar{Q}(0) \sigma_{\alpha\beta} Q(0)$ is not invariant. For this reason the constant $f_T(\mu)$ depends on scale μ as

$$f_T(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{4}{3b_0}} f_T(\mu_0), \quad (5)$$

but the constant f_L does not depend on scale.

LCWFs $\phi_{L,T}(x, \mu)$ can be expanded [12] in Gegenbauer polynomials $C_n^{3/2}(\xi)$ as follows

$$\phi_{L,T}(\xi, \mu) = \frac{3}{4} (1 - \xi^2) \left[1 + \sum_{n=2,4,\dots} a_n^{L,T}(\mu) C_n^{3/2}(\xi) \right]. \quad (6)$$

At leading logarithmic accuracy the coefficients $a_n^{L,T}$ are renormalized multiplicatively

$$a_n^{L,T}(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\epsilon_n^{L,T}/b_0} a_n^{L,T}(\mu_0), \quad (7)$$

where

$$\begin{aligned} \epsilon_n^L &= \frac{4}{3} \left(1 - \frac{2}{(n+1)(n+2)} + 4 \sum_{j=2}^{n+1} \frac{1}{j} \right), \\ \epsilon_n^T &= \frac{4}{3} \left(4 \sum_{j=2}^{n+1} \frac{1}{j} \right), \quad b_0 = 11 - \frac{2}{3} n_f. \end{aligned} \quad (8)$$

It should be noted here that conformal expansions (6) are solution of Bethe-Salpeter equation with one gluon exchange kernel [11].

From equations (6)-(8) it is not difficult to see that at infinitely large energy scale $\mu \rightarrow \infty$ LCWFs $\phi_{L,T}(\xi, \mu)$ tends to the asymptotic form $\phi_{as}(\xi) = 3/4(1 - \xi^2)$. But at energy scales accessible at current experiments the LCWFs

$\phi_{T,L}(\xi, \mu)$ are far from their asymptotic forms. The main goal of this paper is to calculate the LCWFs $\phi_{L,T}(\xi, \mu)$ of J/Ψ meson. These LCWFs will be parameterized by their moments at some scale:

$$\langle \xi_{L,T}^n \rangle_\mu = \int_{-1}^1 d\xi \xi^n \phi_{L,T}(\xi, \mu). \quad (9)$$

It is worth noting that since J/Ψ meson has negative charge parity the LCWFs $\phi_{L,T}(\xi, \mu)$ are ξ -even. Thus all odd moments $\langle \xi_{L,T}^{2k+1} \rangle$ equal zero and one needs to calculate only even moments.

Below the following formulas will be used in our calculation

$$\begin{aligned} \langle 0 | \bar{Q} \gamma_\nu (i z^\sigma \overleftrightarrow{D}_\sigma)^n Q | J/\Psi(\epsilon_{\lambda=0}, p) \rangle &= f_L p_\nu (z p)^n \langle \xi_L^n \rangle, \\ \langle 0 | \bar{Q} \sigma_{\mu\nu} (i z^\sigma \overleftrightarrow{D}_\sigma)^n Q | J/\Psi(\epsilon_{\lambda=\pm 1}, p) \rangle &= f_T (\epsilon_\mu p_\nu - \epsilon_\nu p_\mu) (z p)^n \langle \xi_T^n \rangle, \end{aligned} \quad (10)$$

where

$$\overleftrightarrow{D} = \overrightarrow{D} - \overleftarrow{D}, \quad \overrightarrow{D} = \overrightarrow{\partial} - i g B^a (\lambda^a / 2). \quad (11)$$

These formulas can be obtained if one expands both sides of equations (1).

III. THE MOMENTS IN THE FRAMEWORK OF POTENTIAL MODELS.

In potential models charmonium mesons are considered as a quark-antiquark system bounded by some static potential. These models allow one to understand many properties of charmonium mesons. For instance, the spectrum of charmonium family can be well reproduced in the framework of potential models [18]. Due to this success one can hope that potential models can be applied to the calculation of charmonium equal time wave functions.

Having equal time wave function of J/Ψ meson in momentum space $\psi(\mathbf{k})$ one can apply Brodsky-Huang-Lepage (BHL) [19] procedure and get the LCWFs of leading twist $\phi_{L,T}(\xi, \mu)$ using the following rule:

$$\phi_{L,T}(\xi, \mu) \sim \int^{\mathbf{k}_\perp^2 < \mu^2} d^2 k_\perp \psi_c(x, \mathbf{k}_\perp), \quad (12)$$

where $\psi_c(x, \mathbf{k}_\perp)$ can be obtained from $\psi(\mathbf{k})$ after the substitution

$$\mathbf{k}_\perp \rightarrow \mathbf{k}_\perp, \quad k_z \rightarrow (x_1 - x_2) \frac{M_0}{2}, \quad M_0^2 = \frac{M_c^2 + \mathbf{k}_\perp^2}{x_1 x_2}. \quad (13)$$

Here M_c is a quark mass in potential model. In this paper equal time wave function $\psi(\mathbf{k})$ will be calculated in the framework of the potential models with Buchmuller-Tye [20] and Cornell potentials [21]. The parameters of Buchmuller-Tye potential model were taken from paper [20]. For Cornell potential $V(r) = -k/r + r/a^2$ the calculation was carried out with the following set of parameters: $k = 0.61$, $a = 2.38 \text{ GeV}^{-1}$, $M_c = 1.84 \text{ GeV}$ [22].

In paper [17] the moments of leading twist LCWF of η_c meson were calculated within potential models with these potentials. At leading order approximation in relative velocity there is no difference between equal time wave functions of η_c and J/Ψ mesons. In what follows the moments obtained in paper [17] for the leading twist LCWF of η_c meson equal to the moments of LCWFs $\phi_{L,T}(\xi, \mu)$ of J/Ψ meson. Within this approximation there is no difference between $\phi_L(\xi, \mu)$ and $\phi_T(\xi, \mu)$.

It is worth noting that in paper [23] the relations between the light cone wave functions and equal time wave functions of charmonium mesons in the rest frame were derived. The procedure proposed in paper [23] is similar to BHL with the difference: in formula (12) one must make the substitution $d^2 k_\perp \rightarrow d^2 k_\perp \sqrt{\mathbf{k}^2 + m_c^2} / (4m_c x_1 x_2)$. But this substitution was derived at leading order approximation in relative velocity of quark-antiquark motion inside the charmonium. At this approximation $\mathbf{k}^2 \sim O(v^2)$, $4x_1 x_2 \sim 1 + O(v^2)$ and the substitution amounts to $d^2 k_\perp \rightarrow d^2 k_\perp (1 + O(v^2))$. Thus at leading order approximation applied in [23] these two approaches coincide.

The results of paper [17] are presented in Table I (see this paper for details). In second and third columns the moments calculated in the framework of Buchmuller-Tye and Cornell models are presented. It should be noted that the moments from Table I were calculated at scale $\mu \sim 1.5 \text{ GeV}$. It is seen that there is good agreement between these two models.

It should be noted here that the larger the power of the moment the larger contribution from the end point regions ($x \sim 0$ and $x \sim 1$) it gets. From formulas (13) one sees that the motion of quark-antiquark pair in this region is relativistic and cannot be considered reliably in the framework of potential models. Thus it is not possible to calculate higher moments within the potential models. Due to this fact the calculation of the moments has been restricted by few first moments.

$\langle \xi^n \rangle$	Buchmuller-Tye model [20]	Cornell model [21]	NRQCD [24]	QCD sum rules $\phi_L(\xi, \mu)$	QCD sum rules $\phi_T(\xi, \mu)$
$n = 2$	0.086	0.084	0.075 ± 0.011	0.070 ± 0.007	0.072 ± 0.007
$n = 4$	0.020	0.019	0.010 ± 0.003	0.012 ± 0.002	0.012 ± 0.002
$n = 6$	0.0066	0.0066	0.0017 ± 0.0007	0.0031 ± 0.0008	0.0033 ± 0.0007

TABLE I: The moments of LCWFs $\phi_L(\xi, \mu)$, $\phi_T(\xi, \mu)$ obtained within different approaches. In the second and third columns the moments calculated in the framework of Buchmuller-Tye and Cornell potential models are presented. In the fourth column NRQCD predictions for the moments are presented. In last two columns the results obtained within QCD sum rules are shown.

IV. THE MOMENTS IN THE FRAMEWORK OF NRQCD.

In paper [17] the relations that allow one to connect the moments of leading twist LCWF of η_c meson with NRQCD matrix elements were derived

$$\begin{aligned}
\langle \xi^2 \rangle &= \frac{1}{3M_c^2} \frac{\langle 0 | \chi^+ (i\vec{D})^2 \psi | \eta_c \rangle}{\langle 0 | \chi^+ \psi | \eta_c \rangle} = \frac{\langle v^2 \rangle}{3}, \\
\langle \xi^4 \rangle &= \frac{1}{5M_c^4} \frac{\langle 0 | \chi^+ (i\vec{D})^4 \psi | \eta_c \rangle}{\langle 0 | \chi^+ \psi | \eta_c \rangle} = \frac{\langle v^4 \rangle}{5}, \\
\langle \xi^6 \rangle &= \frac{1}{7M_c^6} \frac{\langle 0 | \chi^+ (i\vec{D})^6 \psi | \eta_c \rangle}{\langle 0 | \chi^+ \psi | \eta_c \rangle} = \frac{\langle v^6 \rangle}{7},
\end{aligned} \tag{14}$$

where ψ and χ^+ are Pauli spinor fields that annihilate a quark and an antiquark respectively, $M_c = M_{J/\Psi}/2$, The moments are defined at scale $\mu \sim M_c$.

These relations were derived at leading order approximation in relative velocity. However, as it was noted above at this approximation there is no difference between η_c and J/Ψ mesons. Moreover, there is no difference between LCWFs $\phi_L(\xi, \mu)$ and $\phi_T(\xi, \mu)$. So the values for the moments of LCWFs $\phi_L(\xi, \mu)$, $\phi_T(\xi, \mu)$ can be taken from paper [17].

The results of the calculation of the moments are presented in the fourth column of Table I. The central values of the moments and the errors due to the model uncertainty have been calculated according to the approach proposed in paper [24]. In addition to the error shown in Table I there is an uncertainty due to higher order v corrections. For the second moment one can expect that this error is about 30%. For higher moments this error is larger.

It is seen from Table I that NRQCD predictions for the second and the fourth moments are in good agreement with potential model and there is disagreement for the moment $\langle \xi^6 \rangle$ between these two approaches. The cause of this disagreement is the fact noted above: due to the large contribution of relativistic motion of quark-antiquark pair inside quarkonium it is not possible to apply both approaches for higher moments. So one can expect that both approaches can be used for the estimation of the values of the second and the fourth moments. The predictions for the sixth and higher moments become unreliable.

V. THE MOMENTS IN THE FRAMEWORK OF QCD SUM RULES.

In this section QCD sum rules [25, 26] will be applied to the calculation of the moments of LCWFs $\phi_L(\xi, \mu)$ and $\phi_T(\xi, \mu)$ [12, 27]. First let us consider LCWF $\phi_L(\xi, \mu)$. To calculate the moments of this LCWF one should consider two-point correlator:

$$\begin{aligned}
\Pi_L(z, q) &= i \int d^4x e^{iqx} \langle 0 | T J_0(x) J_n(0) | 0 \rangle = (zq)^{n+2} \Pi_L(q^2), \\
J_0(x) &= \bar{Q}(x) \hat{z} Q(x), \quad J_n(0) = \bar{Q}(0) \hat{z} (i z^\rho \vec{D}_\rho)^n Q(0), \quad z^2 = 0.
\end{aligned} \tag{15}$$

It is not difficult to obtain sum rules for this correlator (for details see paper [17]).

$$\frac{f_L^2 \langle \xi_L^n \rangle}{(M_{J/\Psi}^2 + Q^2)^{m+1}} = \frac{1}{\pi} \int_{4m_c^2}^{\infty} ds \frac{\text{Im} \Pi_{\text{pert}}(s)}{(s + Q^2)^{m+1}} + \Pi_{\text{npert}}^{(m)}(Q^2), \tag{16}$$

where perturbative and nonperturbative contributions to sum rules $\text{Im}\Pi_{\text{pert}}(s)$, $\Pi_{\text{npert}}^{(m)}(Q^2)$ can be written as

$$\text{Im}\Pi_{\text{pert}}(s) = \frac{3}{8\pi} v^{n+1} \left(\frac{1}{n+1} - \frac{v^2}{n+3} \right), \quad v^2 = 1 - \frac{4m_c^2}{s}, \quad (17)$$

$$\begin{aligned} \Pi_{\text{npert}}^{(m)}(Q^2) &= \Pi_1^{(m)}(Q^2) + \Pi_2^{(m)}(Q^2) + \Pi_3^{(m)}(Q^2), \\ \Pi_1^{(m)}(Q^2) &= \frac{\langle \alpha_s G^2 \rangle}{24\pi} (m+1) \int_{-1}^1 d\xi \left(\xi^n + \frac{n(n-1)}{4} \xi^{n-2} (1-\xi^2) \right) \frac{(1-\xi^2)^{m+2}}{(4m_c^2 + Q^2(1-\xi^2))^{m+2}}, \\ \Pi_2^{(m)}(Q^2) &= -\frac{\langle \alpha_s G^2 \rangle}{6\pi} m_c^2 (m^2 + 3m + 2) \int_{-1}^1 d\xi \xi^n (1+3\xi^2) \frac{(1-\xi^2)^{m+1}}{(4m_c^2 + Q^2(1-\xi^2))^{m+3}}, \\ \Pi_3^{(m)}(Q^2) &= \frac{\langle \alpha_s G^2 \rangle}{384\pi} (n^2 - n)(m+1) \int_{-1}^1 d\xi \xi^{n-2} \frac{(1-\xi^2)^{m+3}}{(4m_c^2 + Q^2(1-\xi^2))^{m+2}}. \end{aligned} \quad (18)$$

Here $Q^2 = -q^2$, m_c and $\langle \alpha_s G^2 \rangle$ are parameters of QCD sum rules.

To calculate the moments of LCWF $\phi_T(\xi, \mu)$ one should consider the correlator:

$$\begin{aligned} \Pi_T(z, q) &= i \int d^4x e^{iqx} \langle 0 | T J_\mu(x) J_\mu^\mu(0) | 0 \rangle = (zq)^{n+2} \Pi_T(q^2), \\ J_\mu(x) &= \bar{Q}(x) (\sigma_{\mu\nu} z^\nu) Q(x), \quad J_\mu^\mu(0) = \bar{Q}(0) (\sigma^{\mu\nu} z_\nu) (i z^\rho \overleftrightarrow{D}_\rho)^n Q(0), \quad z^2 = 0. \end{aligned} \quad (19)$$

The sum rules for this correlator can be written as

$$\frac{f_T^2 \langle \xi_T^n \rangle}{(m_{J/\Psi}^2 + Q^2)^{m+1}} = \frac{1}{\pi} \int_{4m_c^2}^{\infty} ds \frac{\text{Im}\Pi_{\text{pert}}(s)}{(s + Q^2)^{m+1}} + \Pi_{\text{npert}}^{(m)}(Q^2), \quad (20)$$

where perturbative and nonperturbative contributions to sum rules (20) are given by expressions (17), (18) except that the expression for $\Pi_1^{(m)}(Q^2)$ should be replaced by

$$\Pi_1^{(m)}(Q^2) = \frac{\langle \alpha_s G^2 \rangle}{24\pi} (m+1) \int_{-1}^1 d\xi \left(-\xi^n + \frac{n(n-1)}{4} \xi^{n-2} (1-\xi^2) \right) \frac{(1-\xi^2)^{m+2}}{(4m_c^2 + Q^2(1-\xi^2))^{m+2}}. \quad (21)$$

In the original paper [26] the method QCD sum rules was applied at $Q^2 = 0$. However, as it was shown in paper [28], there is large contribution of higher dimensional operators at $Q^2 = 0$ which grows rapidly with m . To suppress this contribution sum rules (16), (20) will be applied at $Q^2 = 4m_c^2$.

In the numerical analysis of QCD sum rules the values of parameters m_c and $\langle \alpha_s G^2 / \pi \rangle$ will be taken from paper [28]:

$$m_c = 1.24 \pm 0.02 \text{ GeV}, \quad \langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012 \pm 30\% \text{ GeV}^4. \quad (22)$$

First sum rules (16), (20) will be applied to the calculation of the constants $f_{L,T}^2$. It is not difficult to express the constants $f_{L,T}^2$ from equations (16), (20) at $n = 0$ as functions of m . For too small values of m ($m < m_1$) there are large contributions from higher resonances and continuum which spoil sum rules (16), (20). Although for $m \gg m_1$ these contributions are strongly suppressed, it is not possible to apply sum rules for too large m ($m > m_2$) since the contribution arising from higher dimensional vacuum condensates rapidly grows with m and invalidates approximation of this paper. If $m_1 < m_2$ there is some region of applicability of sum rules (16), (20) $[m_1, m_2]$ where both resonance and higher dimensional vacuum condensates contributions are not too large. Within this region $f_{L,T}^2$ as a functions of m vary slowly and one can determine the values of these constants. Applying approach described above one gets

$$\begin{aligned} f_L^2 &= 0.170 \pm 0.002 \pm 0.004 \pm 0.016 \text{ GeV}^2, \\ f_T^2 &= 0.167 \pm 0.002 \pm 0.003 \pm 0.016 \text{ GeV}^2. \end{aligned} \quad (23)$$

The first error in (23) corresponds the variation of the constants $f_{L,T}^2$ within the region of stability. The second and the third errors in (23) correspond to the variation of the gluon condensate $\langle \alpha_s G^2 \rangle$ and the mass m_c within ranges (22). From the results (23) one sees that the main errors in determination of the constants $f_{L,T}^2$ result from the

variation of the parameter m_c . This fact represents well known property: high sensitivity of QCD sum rules to the mass parameter m_c .

Next let us consider the second moments $\langle \xi_{L,T}^2 \rangle$ in the framework of QCD sum rules. One way to find the values of $\langle \xi_{L,T}^2 \rangle$ is to determine the values of $f_{L,T}^2 \langle \xi_{L,T}^2 \rangle$ from sum rules (16), (20) at $n = 2$ and then extract $\langle \xi_{L,T}^2 \rangle$. However, as it was noted above, this approach suffers from high sensitivity of right side of equations (16), (20) to the variation of the parameter m_c . Moreover, the quantities $f_{L,T}^2 \langle \xi_{L,T}^2 \rangle$ include not only the errors in determination of $\langle \xi_{L,T}^2 \rangle$, but also the errors in $f_{L,T}^2$. To remove these disadvantages the ratios of sum rules at $n = 2$ and $n = 0$: $f_{L,T}^2 \langle \xi_{L,T}^2 \rangle / f_{L,T}^2$ will be considered. The moments $\langle \xi_{L,T}^4 \rangle, \langle \xi_{L,T}^6 \rangle$ will be considered analogously. Applying standard procedure one gets the moments of LCWF $\phi_L(\xi, \mu)$:

$$\begin{aligned} \langle \xi_L^2 \rangle &= 0.070 \pm 0.002 \pm 0.007 \pm 0.002, \\ \langle \xi_L^4 \rangle &= 0.012 \pm 0.001 \pm 0.002 \pm 0.001, \\ \langle \xi_L^6 \rangle &= 0.0031 \pm 0.0002 \pm 0.0008 \pm 0.0002, \end{aligned} \quad (24)$$

and moments of LCWF $\phi_T(\xi, \mu)$

$$\begin{aligned} \langle \xi_T^2 \rangle &= 0.072 \pm 0.002 \pm 0.007 \pm 0.002, \\ \langle \xi_T^4 \rangle &= 0.012 \pm 0.001 \pm 0.002 \pm 0.001, \\ \langle \xi_T^6 \rangle &= 0.0033 \pm 0.0002 \pm 0.0007 \pm 0.0003. \end{aligned} \quad (25)$$

The first error in (24), (25) corresponds the variation within the region of stability. The second and the third errors in (24), (25) correspond to the variation of the gluon condensate $\langle \alpha_s G^2 \rangle$ and the mass m_c within ranges (22). It is seen that, as one expected, the sensitivity of the ratios $f_{L,T}^2 \langle \xi_{L,T}^2 \rangle / f_{L,T}^2$ to the variation of m_c is rather low. The main source of uncertainty is the variation of gluon condensate $\langle \alpha_s G^2 \rangle$. In the fifth and sixth columns of Table I results (24), (25) are presented. The errors in Table I correspond to the main source of uncertainty — the variation of gluon condensate $\langle \alpha_s G^2 \rangle$.

In the calculations of the correlators (15), (19) characteristic virtuality of quark is $\sim (4m_c^2 + Q^2)/m \sim m_c^2$. So the values of the moments (24), (25) are defined at scale $\sim m_c^2$.

From Table I it is seen that the larger the number of the moment n the larger the uncertainty due to the variation of vacuum gluon condensate. This property is a consequence of the fact that the role of power corrections in the sum rules (16), (20) grows with n . From this one can conclude that there is considerable nonperturbative contribution to the moments $\langle \xi_{L,T}^n \rangle$ with large n what means that nonperturbative effects are very important in relativistic motion of quark-antiquark pair inside the meson. The second important contribution to QCD sum rules (16), (20) at large n is QCD radiative corrections to perturbative part $\Pi_{\text{pert}}(Q^2)$. Unfortunately today one does not know the expression for these corrections and for this reason they are not included to sum rules (16), (20). One can only say that these corrections grow with n and, probably, the size of radiative corrections to the ratios $f_{L,T}^2 \langle \xi_{L,T}^n \rangle / f_{L,T}^2$ is not too big for not too large n . Thus one can expect that QCD radiative corrections will not change dramatically the results for the moments $n = 2$ and $n = 4$. But the radiative corrections to $\langle \xi_{L,T}^6 \rangle$ may be important.

It is interesting to compare the moments of leading twist LCWF of η_c meson calculated in paper [17]

$$\begin{aligned} \langle \xi_{\eta_c}^2 \rangle &= 0.070 \pm 0.002 \pm 0.007 \pm 0.003, \\ \langle \xi_{\eta_c}^4 \rangle &= 0.012 \pm 0.001 \pm 0.002 \pm 0.001, \\ \langle \xi_{\eta_c}^6 \rangle &= 0.0032 \pm 0.0002 \pm 0.0009 \pm 0.0003, \end{aligned} \quad (26)$$

with the results of this section. It is seen that there is no significant difference between the moments of leading twist LCWF of η_c meson and the moments of LCWFs $\phi_L(\xi, \mu), \phi_T(\xi, \mu)$. The difference is within the error of the calculation.

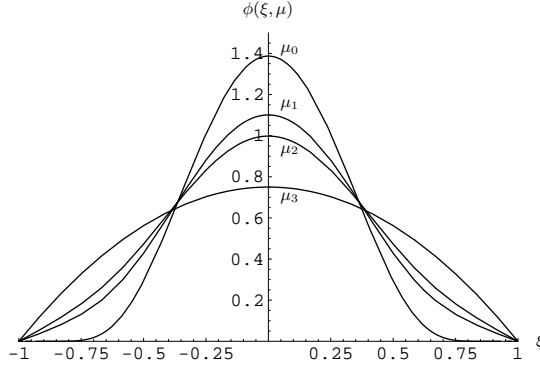


Fig.1: The LCWF ϕ_L (29) at scales $\mu_0 = 1.2$ GeV, $\mu_1 = 10$ GeV, $\mu_2 = 100$ GeV, $\mu_3 = \infty$.

VI. THE MODEL FOR THE LCWFs OF J/Ψ MESON.

In paper [17] the model of leading twist LCWF of η_c meson was proposed:

$$\phi(\xi, \mu = \mu_0) = c(\beta)(1 - \xi^2)\exp\left(-\frac{\beta}{1 - \xi^2}\right), \quad (27)$$

where $c(\beta)$ is a normalization coefficient, the constant $\beta = 3.8 \pm 0.7$, the scale $\mu_0 = 1.2$ GeV. This function allows one to reproduce the results (26) with rather good accuracy. The moments of this wave function are

$$\begin{aligned} \langle \xi^2 \rangle &= 0.070 \pm 0.007, \\ \langle \xi^4 \rangle &= 0.012 \pm 0.002, \\ \langle \xi^6 \rangle &= 0.0030 \pm 0.0009. \end{aligned} \quad (28)$$

At central value $\beta = 3.8$ the constant $c(\beta) \simeq 62$.

As it was noted in the previous section the accuracy of the calculation does not allow one to distinguish leading twist LCWF of η_c meson from LCWFs of J/Ψ meson. For this reason the model (27) will be used for the LCWFs $\phi_L(\xi, \mu = \mu_0)$, $\phi_T(\xi, \mu = \mu_0)$

$$\phi_L(\xi, \mu = \mu_0) = \phi_T(\xi, \mu = \mu_0) = \phi(\xi, \mu = \mu_0). \quad (29)$$

In this expression the functions $\phi_L(\xi, \mu = \mu_0)$, $\phi_T(\xi, \mu = \mu_0)$ are defined at scale $\mu = \mu_0$. It is not difficult to calculate these functions at any scale $\mu > \mu_0$ using conformal expansions (6). The LCWFs $\phi_L(\xi, \mu)$ at scales $\mu_0 = 1.2$ GeV, $\mu_1 = 10$ GeV, $\mu_2 = 100$ GeV, $\mu_3 = \infty$ are shown in Fig. 1. The moments of LCWFs $\phi_L(\xi, \mu)$ at scales $\mu_0 = 1.2$ GeV, $\mu_1 = 10$ GeV, $\mu_2 = 100$ GeV, $\mu_3 = \infty$ are presented in second, third, fourth and fifth columns of Table II. The plot and the moments of LCWF $\phi_T(\xi, \mu)$ will not be shown since this function practically does not deviate from $\phi_L(\xi, \mu)$.

As was noted in paper [17] model (29) has some interesting properties. For instance, let us consider the LCWF $\phi_L(\xi, \mu)$ (similar consideration can be done for $\phi_T(\xi, \mu)$). From conformal expansion (6) one can derive the expressions that determine the evolution of the moments:

$$\begin{aligned} \langle \xi_L^2 \rangle_\mu &= \frac{1}{5} + a_2^L(\mu) \frac{12}{35}, \\ \langle \xi_L^4 \rangle_\mu &= \frac{3}{35} + a_2^L(\mu) \frac{8}{35} + a_4^L(\mu) \frac{8}{77}, \\ \langle \xi_L^6 \rangle_\mu &= \frac{1}{21} + a_2^L(\mu) \frac{12}{77} + a_4^L(\mu) \frac{120}{1001} + a_6^L(\mu) \frac{64}{2145}. \end{aligned} \quad (30)$$

Similar relations can be found for any moment. Further let us consider the expression for the second moment $\langle \xi_L^2 \rangle$. In this paper the value $\langle \xi_L^2 \rangle$ has been found with some error at scale $\mu = \mu_0$. This means that the value of the coefficient $a_2^L(\mu = \mu_0)$ was found with some error. The coefficient a_2^L decreases as scale increases. So the error in a_2^L and consequently in $\langle \xi_L^2 \rangle$ decreases as scale increases. At infinitely large scale there is no error in $\langle \xi_L^2 \rangle$ at all. The calculations show that the error 10% in $\langle \xi_L^2 \rangle$ at scale $\mu = \mu_0$ decreases to 4% at scale $\mu = 10$ GeV. Applying relations (30) it is not difficult to show that similar improvement of the accuracy happens for higher moments. The

improvement of the accuracy allows one to expect that model (29) at larger scales will be rather good even if QCD radiative corrections to results (24), (25) are large.

From Fig. 1 one sees that LCWF at scale $\mu = \mu_0$ practically vanishes in the regions $0.75 < |\xi| < 1$. In this region the motion of quark-antiquark pair is relativistic and vanishing of LCWF in this region means that at scale $\mu = \mu_0$ charmonium can be considered as a nonrelativistic bound state of quark-antiquark pair with characteristic velocity $v^2 \sim 1/\beta \sim 0.3$. Further let us regard the function $\phi_L(\xi, \mu = \mu_0)$ as a conformal expansion (6). To get considerable suppression of the LCWF in the region $0.75 < |\xi| < 1$ one should require fine tuning of the coefficients of conformal expansion $a_n^L(\mu = \mu_0)$. The evolution of the constants a_n^L (especially with large n) near $\mu = \mu_0$ is rather rapid (see formulas (7) and (8)) and if there is fine tuning of the constants at scale $\mu = \mu_0$ this fine tuning will be rapidly broken at larger scales. This property is well seen in Table II and Fig. 1. From Fig. 1 it is seen that there is relativistic tail in the region $0.75 < |\xi| < 1$ for scales $\mu = 10, 100$ GeV which is absent at scale $\mu = \mu_0$. Evidently this tail cannot be regarded in the framework of NRQCD. This means that, strictly speaking, at some scale charmonium can not be considered as nonrelativistic particle and the application of NRQCD to the production of charmonium at large scales may lead to large error. Although in the above arguments the model of LCWF (29) was used it is not difficult to understand that the main conclusion is model independent.

According to the velocity scaling rule [1] the moments $\langle \xi_L^n \rangle$ of LCWF depend on relative velocity as $\sim v^n$. It is not difficult to show that the moments of LCWF (29) satisfy these rules. Now let us consider the expressions that allows one to connect the coefficients of conformal expansion a_n^L with the moments $\langle \xi_L^n \rangle$. These expressions for the moments $\langle \xi_L^2 \rangle, \langle \xi_L^4 \rangle, \langle \xi_L^6 \rangle$ are given by formulas (30). It causes no difficulties to find similar expressions for any moment. From expressions (30) one sees that to get velocity scaling rules: $\langle \xi_L^n \rangle \sim v^n$ at some scale one should require fine tuning of the coefficients a_n^L at this scale. But, as was already noted above, if there is fine tuning of the coefficients at some scale this fine tuning will be broken at larger scales. From this one can conclude that velocity scaling rules are broken at large scales.

Consider the moments of LCWF (29) at infinite scale. It is not difficult to find that

$$\langle \xi_{L,T}^n \rangle_{\mu=\infty} = \frac{3}{(n+1)(n+3)}. \quad (31)$$

From last equation one can find that $\langle \xi_{L,T}^n \rangle$ does not scale as v^n as velocity scale rules [1] require. Thus scaling rules obtained in paper [1] are broken for asymptotic function. Actually one does not need to set the scale μ to infinity to break these rules. For any scale $\mu > \mu_0$ there is a number n_0 for which the moments $\langle \xi_{L,T}^n \rangle$, $n > n_0$ violate velocity scaling rules. This property is a consequence of the following fact: beginning from some $n = n_0$ the contribution of the relativistic tail of LCWF, that appears at scales $\mu > \mu_0$, to the moments becomes considerable.

The amplitude T of any hard process with charmonium meson production can be written as a convolution of LCWF $\Phi(\xi)$ with hard kernel $H(\xi)$ of the process. If one expands this kernel over ξ and substitute this expansion to the amplitude T one gets the results:

$$T = \int d\xi H(\xi) \Phi(\xi) = \sum_n \frac{H^{(n)}(0)}{n!} \langle \xi^n \rangle. \quad (32)$$

If one takes the scale $\mu \sim \mu_0$ in formula (32), than moments $\langle \xi^n \rangle$ scale according to the velocity scaling rules $\sim v^n$ and one gets usual NRQCD expansion of the amplitude. However due to the presence of the scale of the hard process $\mu_h \gg \mu_0$ there appear large logarithms $\log \mu_h/\mu_0$ which spoil NRQCD expansion (32). To remove this large logarithms one should take $\mu \sim \mu_h$. But at large scales velocity scaling rules are broken and application of NRQCD is questionable.

In papers [13, 23, 29] it was proposed different models of LCWFs of J/Ψ and η_c mesons. It is interesting to compare the models proposed in these papers with model (29). Such comparison was done in paper [17] and it will not be discussed in this paper.

VII. CONCLUSION.

In this paper the moments of leading twist light cone wave functions (LCWF) of J/Ψ meson have been calculated within three approaches. In the first approach Buchmuller-Tye and Cornell potential models were applied to the calculation of the moments of LCWFs. In the second approach the moments of LCWFs were calculated in the framework of NRQCD. In the third approach the method QCD sum rules was applied to the calculation of the moments. The results obtained within these three approaches are in good agreement with each other for the second moment $\langle \xi^2 \rangle$. There is a little disagreement between the predictions for the fourth moment $\langle \xi^4 \rangle$. The disagreement

$\langle \xi^n \rangle$	$\phi(\xi, \mu_0 = 1.2 \text{ GeV})$	$\phi(\xi, \mu_1 = 10 \text{ GeV})$	$\phi(\xi, \mu_2 = 100 \text{ GeV})$	$\phi(\xi, \mu_3 = \infty)$
$n = 2$	0.070	0.12	0.14	0.20
$n = 4$	0.012	0.040	0.052	0.086
$n = 6$	0.0032	0.019	0.026	0.048

TABLE II: The moments of LCWF (29) proposed in this paper at scales $\mu_0 = 1.2 \text{ GeV}$, $\mu_1 = 10 \text{ GeV}$, $\mu_2 = 100 \text{ GeV}$, $\mu_3 = \infty$ are presented in second, third, fourth and fifth columns.

between the approaches becomes dramatic for the sixth moment $\langle \xi^6 \rangle$. The cause of this disagreement consists in the considerable contribution of relativistic motion of quark-antiquark pair inside J/Ψ meson to higher moments which cannot be regarded reliably in the framework of potential models and NRQCD. The approach based on QCD sum rules is more reliable, especially for higher moments since it does not consider J/Ψ -meson as a nonrelativistic object. The main problem of QCD sum rules is that since there is no expressions of radiative corrections to sum rules one does not know the size these corrections. However, one can expect that QCD radiative corrections will not change the results for the moments $n = 2$ and $n = 4$ dramatically. As to the sixth moment, the contribution the QCD radiative corrections in this case may be important.

The moments of leading twist LCWFs of J/Ψ meson have been compared with the moments of LCWF of η_c . It was found no significant difference between the moments of leading twist LCWF of η_c meson and the moments of LCWFs $\phi_L(\xi, \mu)$, $\phi_T(\xi, \mu)$. The difference is within the error of the calculation. For this reason the model of LCWF of η_c meson was taken as a model for leading twist LCWFs $\phi_L(\xi, \mu)$, $\phi_T(\xi, \mu)$ of J/Ψ meson. As it was shown in paper [17] this model has some interesting properties:

1. Due to the evolution (6) the accuracy of the moments obtained within model (29) improves as the scale rises. For instance, if the error in determination of the moment $\langle \xi_{L,T}^2 \rangle$ is 10% at scale $\mu = \mu_0 = 1.2 \text{ GeV}$, at scale $\mu = 10 \text{ GeV}$ the error is 4%. For higher moments the improvement of the accuracy is even better and there is no error at all at infinite scale $\mu = \infty$. The improvement of the accuracy allows one to expect that model (29) will be rather good even after inclusion of the QCD radiative corrections.

2. At scale $\mu \sim \mu_0$ the LCWFs can be considered as wave functions of nonrelativistic object with characteristic width $\sim v^2 \sim 0.3$. Due to the evolution, at larger scales relativistic tail appears. This tail cannot be considered in the framework of NRQCD and, strictly speaking, at these scales J/Ψ meson is not a nonrelativistic object.

3. It was found that due to the presence of high momentum tail in the LCWFs at scales $\mu > \mu_0$ there is violation of velocity scaling rules obtained in paper [1]. More exactly, for any scale $\mu > \mu_0$ there is a number n_0 for which the moments $\langle \xi_{L,T}^n \rangle$, $n > n_0$ violate NRQCD velocity scaling rules.

Actually, the last two properties are properties of real LCWFs of J/Ψ meson.

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